

## DOCUMENT RESUME

ED 449 983

SE 064 508

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TITLE Algebraic Thinking: Implications for Rethinking Pedagogy and Professional Development.  
INSTITUTION Mid-Continent Research for Education and Learning, Aurora, CO.  
SPONS AGENCY Office of Educational Research and Improvement (ED), Washington, DC.  
PUB DATE 2000-11-00  
NOTE 37p.  
CONTRACT #RJ96006101  
AVAILABLE FROM Mid-continent Research for Education and Learning, 2550 S. Parker Road, Suite 500, Aurora, CO 80014-1678. Tel: 303-337-0990; Fax: 303-337-3005; e-mail: info@mcrel.org; Web site: <http://www.mcrel.org>.  
PUB TYPE Reports - Research (143) -- Tests/Questionnaires (160)  
EDRS PRICE MF01/PC02 Plus Postage.  
DESCRIPTORS \*Algebra; Elementary Secondary Education; Mathematics Curriculum; \*Mathematics Instruction; Professional Development; Teaching Methods; \*Thinking Skills

## ABSTRACT

The recent call for "algebra for all" provided not only the impetus for teaching elementary students differently, but also for changing the way teachers are trained. University mathematics courses for pre-service teachers have undergone changes, and professional development sessions for in-service teachers have been created. The Mid-continent Research for Education and Learning (McREL) has responded to the need to reconceptualize algebra for in-service teachers by initiating a professional development series aimed at training teachers, instructional coordinators, and other school personnel at the elementary school level in algebraic thinking. This report describes the Algebraic Thinking series at McREL and its evaluations. The results of two phases and follow-ups of the implications of the series are discussed. The report also discusses political and policy contexts, related curricula, and professional development, while drawing conclusions. (ASK)

**ALGEBRAIC THINKING:  
IMPLICATIONS FOR RETHINKING PEDAGOGY  
AND PROFESSIONAL DEVELOPMENT**

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**AND PROFESSIONAL DEVELOPMENT**

REL Contract #RJ96006101  
2000 Deliverable #66

Submitted to  
Office of Educational Research and Improvement

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November, 2000



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This work was produced in whole or in part with funds from the Office of Educational Research and Improvement (OERI), U.S. Department of Education, under contract #RJ96006101. The content does not necessarily reflect the views of OERI or the Department of Education, nor does mention of trade names, commercial products, or organizations imply endorsement by the federal government.

This work was prepared by Sheila A. Arens and Rhonda D. Meyer under a subcontract with the Mid-continent Research for Education and Learning (McREL). The views and opinions of the authors do not necessarily reflect the views of McREL.

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## INTRODUCTION

Mathematics is often considered a foundation for successful participation in secondary education and is, perhaps, a cornerstone to citizenship and a requisite for employment. To this end, the National Council of Teachers of Mathematics (NCTM) suggests that mathematics should build on the natural curiosity of children, challenge children to explore increasingly sophisticated concepts, and emphasize the necessity of algebraic thinking at the elementary school level.

Despite the fact that most elementary students are still building number and operation sense, current trends in mathematics education suggest that this early stage is an appropriate time to begin introducing *algebraic thinking*, thereby building the foundation for later comprehension of more advanced concepts. Although there is some conceptual confusion about what *algebraic thinking* specifically entails and how to teach children and educators to think in this way, the common thread among proponents seems to be that algebraic thinking is a way of teaching and learning mathematics that enables younger students to be successful as they advance through increasingly sophisticated concepts and ideas. In this sense, algebra is considered globally: rather than consisting of a set of methods or procedures for determining unknowns, it is conceived of as “a way of thinking” with the algorithms of mathematics being presented as a means of finding an answer rather than the end<sup>1</sup>. Learning abstract rules or procedures is not the goal of learning mathematics, but are simply methods or thinking tools to facilitate learning. Furthermore (and in accordance with national standards), algebraic thinking and mathematics in general are seen as problem-solving strategies used by communities of learners — strategies that often afford opportunities for reaching multiple solutions and utilizing multiple processes to do so.

The recent call for “algebra for all” provided not only the impetus for teaching elementary students differently, but also for changing the way teachers are trained. University mathematics courses for pre-service teachers have undergone changes, and professional development sessions for in-service teachers have been created. The Mid-continent Research for Education and Learning (McREL) has responded to the need to reconceptualize algebra for in-service teachers by initiating a professional development series aimed at training teachers, instructional coordinators, and other school personnel at the elementary school level in algebraic thinking.

## THE ALGEBRAIC THINKING SERIES AT McREL

In McREL’s Algebraic Thinking series, algebraic thinking is understood as using mathematical processes in which students seek patterns and relationships, sort and classify, formulate rules, and discover strategies for solving problems. Through demonstrations of elementary lessons, the Algebraic Thinking series is expected to increase the capacity of participants to foster algebraic thinking among their own students. The Algebraic Thinking series is organized around themes of patterns, functions and relations, modeling, and language.

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<sup>1</sup> Vance, J. (1998). Number operations from an algebraic perspective. *Teaching Children Mathematics*, 4, 282-285.

Over the past four years, the format of Algebraic Thinking has changed in order to address both the needs of potential participants and the logistical difficulties inherent in offering one of a limited number of professional opportunities to an increasing number of teachers. Program characteristics have been modified from providing continuing education units (CEU) for participation, to offering the program as professional development over a five-month period, to offering it as an abbreviated summer session. Although this evaluation focuses on the Summer 2000 series, a brief overview of the earlier series is also provided.

The first Algebraic Thinking series was offered in four 4-hour sessions in January and February of 1997. Fifty-four teachers from 15 public schools and 2 parochial schools participated in these sessions, with 26 receiving CEU through Harris-Stowe State College as well as a stipend. Most teachers who participated in the first session taught grades K–5, although several taught in middle schools (grades 6–8). Feedback regarding these first sessions led the instructor to decide that four-hour blocks were too long to hold teachers' attention. Subsequently, the sessions were shortened.

In the summer of 1997, a subset of 20 teachers from the first group of participants (representing six of the schools) participated in a one-week follow-up to the winter 1997 series. Participants' classrooms were visited, teachers' work with students was observed, and teachers were asked if they needed additional assistance.

Beginning in the fall of 1997, a second series of sessions was offered, consisting of five 2.5- to 3-hour sessions held over a five-month period. Forty teachers from 12 schools (10 of which had not previously participated) attended the sessions. A third series began in the fall of 1998 and was composed of six 2.5- to 3-hour sessions with 17 teachers, seven of whom received credit for their participation. A fourth series (which is the focus of this report) was conducted in the summer of 2000 with 25 teachers at 2 schools (hereafter referred to as School 1 and School 2). Site schools (School 1 and 2) chose to participate in the Algebraic Thinking session. Given that only a limited number of professional development opportunities are provided districtwide in the summer, a number of teachers from outside the site schools also participated.

Since its inception, the Algebraic Thinking series has been offered in a single school district in Missouri, although different schools in the district have opted to participate over the years. The present evaluation focuses on two schools that had not previously participated. Although the student-to-staff ratios and teacher characteristics of the district (in terms of salary, years of experience, and education level) are comparable to the state as a whole, its elementary students' scores on standardized math tests are much lower than the state average (nearly 45% of 4<sup>th</sup> grade students in the district were rated as "progressing" or "nearing proficient" — the lowest achievement categories — compared to 22% statewide) as is its high school graduation rate (district graduation rate is under 50%, whereas the state graduation rate is nearly 80%). The site schools represent two of the poorer schools in the district; over 80% of the students at each school are eligible for free and reduced lunch. And although the student-teacher ratio at School 1 is more favorable than that of the district, School 2 reports a student-teacher ratio nearly 1.5 times that of the district (this school also has nearly 300 more students than School 1).

## **Evaluating the Series**

Initially the evaluation questions guiding the evaluation of the Algebraic Thinking series included the following:

- Were teachers able and did they implement changes to their teaching based on the course material?
- Were there changes in student outcomes (mathematics achievement and student work) for those teachers who implemented changes in classroom practices?
- Were there differences by level of experience in how well teachers implemented new practices?
- Were there differences in level of student accomplishment by level of teacher implementation or experience?
- Does the course in Algebraic Thinking warrant further use in order to increase student success in mathematics and/or improved teacher classroom practices?

These evaluation questions were constructed under the assumption that a core group of teachers would participate in the program over the course of a school semester, be fully exposed to the program, actively engage in thinking and reflecting on their own practice, and have the support they needed within their schools to experiment with a different way of reaching their students. However, Participation at the site schools varied, with some teachers failing to participate in the full series and others participating in only some sessions of the series or exiting sessions early. Also, due to district constraints, the series was not offered during the regular school year but was shortened and offered in the summer. The evaluation of the series was therefore adjusted to include two phases. Phase I includes data from a relatively unsuccessful attempt to evaluate the impact of the first three series of Algebraic Thinking (1997–1998) via surveys of and conversations with teachers; Phase II is a modified case study evaluation of the Summer 2000 series.

## **RESULTS**

### **Phase I (Series 1–3)**

Due to constraints and personnel changes in the school district, the fourth series of Algebraic Thinking scheduled for the 1999–2000 school year was not offered. Recognizing the continuing need to gather evaluative information about the program, it was proposed that a survey be constructed and sent to a sample of 77 participants from series 1, 2 and 3 (1997–1998)<sup>2</sup>. The survey addressed teaching experience (including

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<sup>2</sup> Surveys were targeted at teachers from schools where participation was either fairly extensive among teachers (e.g., a large number of teachers participated) or the teachers' own participation was extensive in terms of time spent in AT and follow-up.



mathematics background and comfort level teaching mathematics concepts), professional development experience, pedagogical practice and curricular choices, and the effectiveness of the Algebraic Thinking series. Our data collection instrument (see Appendix A) was constructed using a modified version of Aiken's Revised Math Attitude Scale (1961) and modified questions from National Council of Teachers of Mathematics (NCTM) questionnaires. Extant materials resulting from previous sessions also were reviewed, including teacher-constructed notebooks to document their use of Algebraic Thinking in the classroom. In addition, we proposed conducting classroom visits and observations of mathematics lessons and in-depth conversations with a randomly selected group of interested survey respondents in order to gain a better understanding of program impact. Packets of surveys (including a letter about the research project addressed to each participant, the survey, and a postage-paid return envelope) were mailed to the principals at participating schools (along with a letter of introduction to the principals). Teachers were offered an incentive (a Whelmer's CD-ROM) if they responded to the survey and informed that a small stipend (\$50) would be provided to any teachers willing to have their classroom visited and to engage in conversations with us about their experiences with the Algebraic Thinking series.

Despite offering an incentive for responding to the survey, as well as numerous other efforts to improve the response rate, only 18 completed surveys were returned (response rate = 23%). Of those teachers responding, only two indicated that they would be willing to have their classroom visited. At this point, we selected several schools where 5 or more teachers had participated in the Algebraic Thinking series and directly contacted the principals to determine if we could visit their schools and speak with former participants (see Appendix B for a copy of the guiding questions for Phase I participant conversations). Although we were able to visit two schools and speak with ten teachers during Phase I site visits, in most cases teachers did not agree to have their classroom observed. The Phase I data were subsequently used as background information for Phase II of the evaluation and are included below.

### **Phase II (Summer 2000)**

Our conversations with the Algebraic Thinking instructor regarding her perceptions of the Summer 2000 participants shortly after the series began led us to believe that one of the sites (School 1) would prove amenable to the program (and possibly receptive to the imposition of the evaluative activities), while the other site (School 2) would possibly prove more difficult. In describing the level of participation at the first session for each site, the instructor noted that the principal and instructional coordinator at School 1 were, themselves, full participants, whereas both of these individuals at School 2 were, at best, partial participants. She commented that the difference between the two schools was like "night and day" in terms of the attitudes of the teachers and the involvement of the administration, suggesting that this stark difference between the two sites could possibly affect teachers' level of involvement in the sessions, commitment to learning, and their ultimate perceptions and utilization of the program. It was with these images of the two site schools that we began our evaluation of the Summer 2000 Algebraic Thinking series.

All of the Summer 2000 series participants were asked to complete an instrument to assess their current level of mathematical understanding, prior experience with mathematics courses, and prior professional development. The survey also solicited

opinions about what was missing in the district-mandated mathematics package that Algebraic Thinking might be able to address. Our data collection instrument questionnaires (see Appendix C) was similar in structure to the Phase I survey; it combined a modified version of Aiken's Revised Math Attitude Scale (1961) and modified questions from NCTM. Nine participants from School 1 and 16 participants from School 2 completed the survey. A review of completed surveys revealed that the two site schools were somewhat similar in terms of teachers' prior knowledge, teaching experience, and backgrounds. However, given that several teachers did not participate in the full series or left sessions early, comparison of these data with follow-up comments and experiences proceeded with caution<sup>3</sup>.

### ***Teaching Experience***

As a group (across both sites), the majority of Summer 2000 participants have had extensive teaching experience, with 68% reporting more than 6 years of teaching experience and 56% reporting more than eleven years of experience. A similar pattern with respect to length of service also was evident in the individual schools. And although the majority of participants classified themselves as self-contained classroom teachers, the group also included teachers of special education, math and science, art, reading, a counselor, and a teacher's aide<sup>4</sup>.

### ***Prior Professional Development***

Across both sites, the majority of participants have spent more than 6 hours in professional development courses in mathematics over the past 2 years (76%) and many reported having spent over 16 hours (44%). Given the district's commitment to implementing a standard, districtwide mathematics curriculum (Investigations) in the past two years, it is not surprising to also learn that many of those hours have been spent learning about the new mathematics curriculum. In fact, all but four of the respondents indicated that they had attended an Investigations workshop within the past two years, and only one of these four was a classroom teacher.

### ***Time Teaching Mathematics***

Summer 2000 participants stated that they spend an average of 71 minutes teaching mathematics on a typical day ( $SD=49$ ; range = 15 to 240). School 1 teachers indicated that they spend 82 minutes per day ( $SD=48$ ; range = 20 to 180) compared to School 2 teachers who spend 65 minutes per day ( $SD=50$ ; range = 15 to 240). Although the variability among all participants and among participants at each school is high, this is quite likely the result of the broad range of grade levels being taught (e.g., preschool teachers indicated they taught mathematics daily for only 15 or 20 minutes).

### ***Series Observations***

During the course of the Summer 2000 series, the evaluation consultants visited both of the sites twice. During the first site visit (late June), we were able to act as "participant-observers" of an Algebraic Thinking session at each school, introduced ourselves and

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<sup>3</sup> Data from teachers from schools outside the site schools are not included in this report.

<sup>4</sup> Additional participants from outside the site schools included a secondary special education teacher and pre-school teachers.

invited teachers to participate in the evaluation project of the Algebraic Thinking series. During the second visit (mid-July), we interviewed participants about the Algebraic Thinking program (11 from School 1, 13 from School 2). Guiding questions focused on such topics as their initial expectations of the program, what they learned, whether they felt they could use what was learned, and what changes might improve the program (the Guiding Questions for this interview appear in Appendix D).

Site interviews with Summer 2000 participants revealed that all participants had been prompted to enroll in the Algebraic Thinking series by their building principal or instructional coordinator. For some participants, this was the only professional development opportunity offered during the summer, which meant their participation was mandatory. Moreover, all schools in the district — in accordance with district requirements outlined in the district improvement plan — are required to use at least 1% of state funding to support professional development activities. For these site schools, the Algebraic Thinking series provided one way to partially meet district guidelines.

Both sites used the first Algebraic Thinking session we observed as a time for teachers to get together, eat lunch, share the day's announcements, and participate in Algebraic Thinking activities. This session was the second of six sessions; thus, participants had already been introduced to the general structure of the sessions.

During this first session observation, participants learned about relations and functions using arrow diagrams, books and stories, and unifix cubes. Participants were shown different strategies for uniting mathematics and literacy and were asked questions that challenged them to explain their solutions and to consider alternate solutions and processes for arriving at solutions.

The rapport that seemed to exist among teachers at each of the sites facilitated open communication and a willingness to engage in activities and make mistakes. It quickly became clear that participants respected the teaching style and knowledge of the Algebraic Thinking instructor. This was evident during sessions when participants asked questions or voiced concerns about teaching mathematics, particularly with respect to how "today's math" and the extensive use of manipulatives to help children discover solutions seemed to disregard what they considered the sine qua non of mathematics: learning "the" facts. The instructor presented balanced answers to their questions. While acknowledging and respecting their claims, she also was able to share her own experience and knowledge of the varied ways children learn as well as the value of thinking about and teaching mathematics in ways that may appear nontraditional. Additional evidence of the rapport between the instructor and the participants and the respect participants afforded the instructor were gleaned from our individual conversations with teachers.

By the end of the Summer 2000 series, School 1 had substantially altered the way teachers participated in the program. During our second site visit (which coincidentally corresponded with the last session day of the program), the Algebraic Thinking instructor spent her day visiting classrooms to teach particular skills (calculator skills) to classes of children enrolled in summer school. Conversations with the instructor indicated that the last three sessions of the Summer 2000 series at School 1 had been altered such that she spent the entire session time presenting lessons in the classrooms with the teacher participants or split session time between classroom demonstration

lessons and Algebraic Thinking lectures. She indicated that school administrators wanted to focus on literacy for the school's professional development in the summer and felt that the Algebraic Thinking series failed to provide such a focus. Despite the instructor's efforts to present Algebraic Thinking as a cross-disciplinary way to think about integrating traditionally discrete subject areas by marrying reading stories and books with mathematics lessons, the school opted to use the contact time with the instructor as a series of individual classroom demonstrations.

Although the instructor's classroom visits at School 1 showed teachers how the concepts covered in the Algebraic Thinking session could be used in their own classrooms, it was not obvious how or whether the concepts actually fit into the summer curriculum. Nor was it obvious that the teachers were learning new ways to think about underlying concepts in mathematics or the actual teaching of mathematics. Unfortunately, the demonstrations seemed to be one way that teachers were able to avoid sitting through sessions that they perceived as not being immediately useful — sessions that the administration itself did not support. The lack of administrative support became more salient to us as we attempted to pursue follow-up site visits and interviews with teachers who had previously made the commitment to be involved in the evaluation at this school.

Participation at School 2 was much more consistent across sessions. At the last session, participants were seated at round tables in the school library, listening to the Algebraic Thinking instructor present ways to encourage algebraic thinking among their students (this session focused on ways in which to use calculators for estimation, "skip counting," and "guess my rule" problems). As was the case with the first session we observed, participants at the second session were eager to make conjectures about possible solutions and appeared enthusiastic about engaging in the activities.

### **Follow-up: Classroom Observations and Conversations**

In October and November 2000, we observed 10 teachers (representing 9 lessons) teach a mathematics lesson (5 lessons in School 1, 4 in School 2). We had made it clear teachers did not have to teach an Algebraic Thinking lesson during our visit. Nonetheless, two of the lessons were clearly derived from the Algebraic Thinking sessions. Several other teachers related that they incorporated parts of Algebraic Thinking into the lessons we observed (though all lessons were typically derived from the district-endorsed mathematics curriculum).

Our observation guides were structured using a modified version of the Classroom Observation Instrument developed at Western Michigan's Science and Mathematics Program Improvement team. Although we eventually were able to visit both site schools, administrators at School 1 were reluctant to allow us to observe teachers (on several occasions administrative staff changed the visit dates or failed to inform teachers we would be there observing). In addition, School 1 administrators insisted that follow-up conversations with teachers occur on the teachers' own time.

Following classroom observations, the 10 teachers participating in the evaluation were contacted via telephone to determine how Algebraic Thinking had influenced their thinking about mathematics and/or their teaching (guiding questions for the follow-up conversation appear in Appendix E). Data from initial conversations with participants,

classroom observations, and follow-up conversations are presented below to support our assertions about the Algebraic Thinking series and about professional development endeavors.

## **Impact**

One of the concerns about professional development courses is whether teachers are actually able to extrapolate what has been presented in the professional development to their own classrooms. Although we did observe several teachers using what they had gleaned from the Algebraic Thinking series in their classrooms, it is difficult to determine whether teachers changed how they think about mathematics or changed their teaching of mathematics. Several evaluation questions (for Phase I: in survey responses and conversations; for Phase II: last session and follow-up phone conversations) focused on whether teachers were able to use what they had learned and how they used it.

## ***Activities***

Despite the length of time that had passed between their participation and our evaluation, a few participants in Phase I of the evaluation were able to provide us with specific examples of Algebraic Thinking activities they have used in their classrooms (e.g., boards, mini-computers, arrow pictures, Venn/string diagrams). One teacher even noted that she has been able to “teach other teachers how to use the activities.” From these conversations (and an initial observation of an Algebraic Thinking lesson), we learned that teachers were able to use some of what they had learned in the sessions they attended. However, as explained below, teachers who wanted to more fully implement Algebraic Thinking in their classrooms met insurmountable policy obstacles both at the school and district levels. These obstacles persisted, influencing the participation and utilization by the Summer 2000 series cohort.

During our initial conversations with Summer 2000 participants, several participants noted that they had already applied some of what they had learned. For instance, a teaching assistant at School 1 stated that she had already seen teachers using what they had learned in their classrooms. Another teacher stated that she had “already used some of the techniques . . . and will be able to use” what she had learned in Algebraic Thinking in the fall (School 2). Across both schools, Summer 2000 participants primarily focused their praise on lessons or activities that they might be able to use in their own classrooms:

The program gives teachers ideas of how to incorporate or implement different activities. (School 2)

[I learned] how to use manipulatives which can translate into my classroom. (School 1)

[I] learned how to use manipulatives better. (School 2)

I have been able to use some of the activities from the Algebraic Thinking Summer sessions during warm-up periods. I've used ideas for manipulatives and the Venn diagrams. (School 2)



Kids really like the Algebraic Thinking activities — hands-on. (School 1)

It has increased the hands-on activities [I use] and helped me with my teaching. I am able to use manipulatives in a different manner. . . . It helped me a lot. (1997–1998 participant)

Other Summer 2000 participants were less satisfied with the activities that were presented, suggesting that they were not getting guidance on how to apply them.

Ideas for activities are provided, but not how to structure the activities. (School 2)

Algebraic Thinking did not provide specific lessons for specific things. . . — teachers have to figure them out. (School 1)

This sense of ambiguity left one teacher feeling that she would not be able to use what she had learned. It left a beginning teacher struggling with how she could find time to use what she had learned: “I might try to use activities if I put them together” (School 2).

Particularly in the lower grades, classroom management issues were profound. Teachers regularly interrupted lessons that we observed to correct a child who was being disruptive by talking to friends or a child who was handling manipulatives before being instructed to do so. Although this certainly changes the flow of the lesson, it draws attention to how time consuming those methods of teaching typically associated with a constructivist classroom can be. For the most part, teachers expressed excitement that they were able to learn about and use manipulatives with their students in the Algebraic Thinking series. However, some teachers remain reluctant to try out different ways of teaching for fear that children would not learn in pairs or groups. Still others expressed concern that providing children with many opportunities to use manipulatives could lead to these objects being used as a crutch. Not all teachers agreed that manipulatives constituted the best (or most appropriate) way to teach mathematics, particularly to children with varied levels of mathematics skill or acumen (participant, 1997–1998 series).

Although teachers at both schools praised the program for its emphasis on engaging children in the learning process via hands-on activities, teachers at School 2 were able to state more explicitly what they had learned from the sessions (both in terms of specific activities and how their thinking about mathematics had been expanded). For instance, a teacher from School 1 stated, “I have been able to use some of what I learned in the Summer sessions, but can’t remember exactly what it was.” This was not surprising given that participants at School 2 were more involved in the Algebraic Thinking sessions and school administrators demonstrated a commitment to the program.

### ***Student Impact***

Although we were able to ask teachers whether and how they thought the Algebraic Thinking series had affected their own students’ learning, because of the short exposure to the Summer 2000 series in conjunction with district mandates regarding curriculum,

examination of such indicators as student test scores (e.g., state-assessment scores, NAEP scores) did not seem prudent. Thus, although teachers may have sensed that Algebraic Thinking (when they were able to use it) made sense to their students or facilitated student learning in some way, drawing conclusions or making claims about the link between a teacher's participation in the series and his or her students is simply unwarranted.

### **Grade Level**

Several teachers across all Algebraic Thinking series voiced concern about how age-, grade-, or developmentally appropriate Algebraic Thinking was. For instance, a 1997–1998 participant stated that although she was convinced that Algebraic Thinking helps students with their abstract thinking/reasoning, she found that the help was particularly relevant for children who already excelled. Moreover, some participants expressed concern about how the program might be adjusted for special education students or students in the lower grades, particularly preschool and kindergarten.

Conversations with participants led us to conclude that when teachers attend professional development courses such as Algebraic Thinking, they focus on what they might be able to take away from it to use immediately with their current students. However, many of the teachers with whom we spoke had changed grade levels since participating in professional development courses. The teachers' focus on immediate utility seems to undermine efforts at presenting Algebraic Thinking as a broad way of conceiving of mathematics — one in which “lessons” or “skills” consist of underlying mathematics concepts that are amenable to adaptation to meet the needs of students from different grade levels and different abilities. This is perhaps best captured by a participant from one of the 1997–1998 series, who remarked: “ Since I've changed classrooms from 5<sup>th</sup> grade to 2<sup>nd</sup> grade, what I learned is not relevant. Are there additional materials?”

In a similar way, several Summer 2000 participants of lower grades expressed disappointment that the material presented in the program would prove too difficult for their students. Several participants stated that the lessons were not geared for younger children and that Algebraic Thinking would be improved if the number of activities or lessons for the lower grades was increased. During our exit interviews and follow-up conversations, teachers of lower levels at both site schools indicated that they needed additional help either adjusting the activities for their students or would like to have simpler concepts presented (along with appropriate lessons and manipulatives). For instance, one preschool teacher suggested that it was important to “consider what the children are capable of and break it down. My students would not know that something was a blue square, but they might be able to understand that it is a square. Or that a yellow triangle is a triangle” (School 2). Another teacher from the same school recommended that the content remain the same but that the program “be scaled up or down based on grade level/age level/level of children.” Other teachers suggested that the Algebraic Thinking series could be improved if it was “broken down more for Kindergartners” or “geared more for lower grades.” A final comment from a School 1 teacher clarifies this sentiment:

Bring it down in language; simplify it. Might have to make it more concrete by using reading as a foundation. It is abstract — students get

answers but they don't understand it. Definitely need manipulatives. More specific examples are needed to help the kids with transfer.

To address the problem of needing more of an emphasis on a particular grade level, one teacher suggested that Algebraic Thinking “may have been easier if individual grades (or groups of similar grades, e.g., 1–2) were together rather than having all grades in the same workshop—not elementary and middle schools” (1997–1998 participant).

The opinion that Algebraic Thinking was not age-, grade-, or developmentally appropriate was not shared by all participants. Indeed, one teacher suggested that the program could be readily adapted to meet the needs of preschoolers. A 4<sup>th</sup> grade teacher provided another dissenting voice, commenting that he “plan[s] to use everything [the instructor has] given us, but [the instructor has] been at a lower level” — indicating that transfer to the classroom would require “scaling the materials up” (School 2). Interestingly one teacher who thought the program would be too difficult for her students commented that what she was teaching from Algebraic Thinking did work with her students:

First I was thinking that my children wouldn't be able to understand arrows and skip counting. But patterns really make sense — I started thinking about it in my classroom. I did get something out of it. A lot of times I try to find easy ways to reach the kids and this provides a different mode of thinking that may work for some. (School 1)

The diversity of comments may suggest a possible relationship between how comfortable teachers are with mathematics and how able they are to think *differently* about mathematics content and pedagogy. That is, teachers who feel adept at mathematics may not find it distressing that they need to adjust the materials to meet the needs of their students. In conversations with the Algebraic Thinking instructor, it was revealed that many of the first sessions of the Summer 2000 series, for example, concentrated on how Algebraic Thinking could be used for the lower grades. Given that participants were exposed to ways to incorporate Algebraic Thinking in the lower grades, it is perplexing that one of the strongest complaints about the program is that it is not simple enough to use for these grades. However, the time it takes to adjust the curriculum or the extra reflection required to make it accessible to particular students (given their unique learning needs) may also play a role in these comments. This is briefly discussed later.

### ***Expectations***

Oftentimes participants' use of what they learn is directly related to how well the presentations correspond to what they expect to learn. Unfortunately, it is not clear that participants entered the Summer 2000 Algebraic Thinking series cognizant of what they would glean for their own teaching.

Several teachers began the Algebraic Thinking series expecting that they would be provided with “consumables” related to lessons or activities they could do in their classes. One participant stated that she expected to receive “handouts of lessons” (School 1) and another teacher “thought it would provide new hands-on activities to use



in the upcoming school year — new concepts” (School 2; participant not from school). Other participants focused on the fact that they did not know what to expect from Algebraic Thinking and enrolled in it primarily because they were required to do so. Several participants from School 1 expected Algebraic Thinking to be similar to — or even repeat — what they had already learned in the Investigations workshop, indicating a possible predilection toward resistance or expectation of boredom. This suggests that there may not have been enough information given to teachers about what to expect from the program. Although conversations among the district liaison, Algebraic Thinking instructor, and school administrators may occur, it is possible that efforts to establish closer link between teachers and these parties is warranted.

Although several teachers did not have specific comments about aspects of the program that they felt were lacking or areas where they thought more assistance was needed, when prompted for specific changes they would recommend or areas of weakness, several teachers stated that they expected to learn more about the connection between mathematics and literacy. Indeed, at both sites, teachers made comments about the cross-disciplinary approach of the Algebraic Thinking. Some suggested that the program would be enhanced by crossing more disciplinary boundaries while others expressed appreciation that the course was not parochial in its approach to mathematics. It is interesting that teachers characterized the program in such divergent ways; this may be partially explained by the site schools’ learning goals for students or it may be partially a function of teachers’ varying conceptions of the purpose of teaching and learning mathematics. For instance, a teacher at School 1 stated that the program needs “more literature tie-ins” and asked whether “there [were] any books” that could be used to supplement particular lessons. This teacher admitted not remembering any specifics of the program (and arguably made this point without recollecting that, in the very least, the second session at her school focused on how to think about links between literacy and algebraic thinking). We must, however, keep in mind that School 1 opted to further abbreviate the Summer 2000 series contact time because administration perceived the goals of their building as being incommensurable with those of the Algebraic Thinking series.

### ***Connection with Existing Curriculum***

Our initial conversations with participants in Phase I of the evaluation made it clear that utilization of Algebraic Thinking had been stymied by district mandates regarding the mathematics curriculum. These participants had enrolled in Algebraic Thinking prior to the districtwide adoption of Investigations and were therefore able to provide some indication of whether and how they were able to implement Algebraic Thinking before Investigations and following Investigations. Summer 2000 participants also provided evidence of the difficulties teachers face when attempting to utilize a way of thinking or teaching that differs from that embraced by the district. Essentially, Algebraic Thinking became a means to support or supplement what was already being taught. This is detailed below.

### ***Rethinking Mathematics and Pedagogy***

One teacher noted that she didn’t really know what to expect from the sessions, except that she’d learn “something about Algebra.” Upon further reflection, this teacher felt that the way she thought about algebra had changed — that the instructor “simplified [algebraic concepts] in a way children can understand and make them more

comfortable.” Speaking of her own teaching philosophy, another participant explained that she believes everyone can learn but that teachers need different ways to teach — they need to provide children ways that they can see it, hear it, touch it (mathematics). In this respect, she felt Algebraic Thinking was good for her students — “it made sense to the children; they could do the activities, tell you what they did, and tell you how they did it.” She also felt that the Algebraic Thinking program provided teachers “different ways to reach other students and showed them that there is more than one way to get to an answer” (1997–1998 participant). Two teachers from School 1 provided additional insights into what they felt they had learned in the Algebraic Thinking series:

I didn't learn a new way of thinking but maybe a new way of teaching, for example with the arrows. (School 1)

I didn't really learn a new way of thinking (since I had already had a lot of math), but maybe I learned a new way of teaching it to kids; teaching logic and reasoning. (School 1)

Although both of these teachers seem to suggest that they learned different ways of approaching the teaching of mathematics in the Summer Algebraic Thinking series, it is interesting that they are able to separate content and pedagogy from their own beliefs about mathematics.

Though participation in the Algebraic Thinking Summer series, one teacher indicated that she had changed her mind about the impact that working in groups could play in student learning. Prior to Algebraic Thinking, she indicated that she only sometimes used cooperative learning groups, not because of classroom management issues but because she feared free-riders would emerge (children who would decide they did not have to work since someone else would do it). However, during post-interviews she explained that Algebraic Thinking had changed her thinking:

In a way [it has changed my thinking] because in the past I was one of those teachers who didn't let children work in teams or groups. Now I feel more comfortable with these kinds of settings — even if the children are not each doing their own work, they can be listening to the other children and learning. I saw that this past summer during the Algebraic Thinking sessions — the teachers worked in groups but it didn't mean that some of us weren't doing the work or weren't learning. My concern with group work was not about classroom management but was about children getting away with not doing the work. (School 2)

And although a number of participants across all series praised Algebraic Thinking for encouraging students to work collaboratively, several teachers also noted that they also value taking more traditional approaches to teaching mathematics — approaches that will “help students listen and focus” (1997–1998 participant). Indeed, one participant from an early series thought that Algebraic Thinking did not provide enough practice for the students, suggesting that although repetition might be boring for the teachers, it provides an opportunity for students to master the material: “We've thrown out rote memorization and thrown out the baby with the bath water — rote memorization has its place.” This teacher also suggested that the district-mandated curriculum, Investigations, suffers from the same ill-fated attempt at changing the way students

learn. What seems clear from our conversations with teachers is that programs such as Algebraic Thinking may serve to support some participants' own teaching philosophies and beliefs about how children learn while it challenges others' beliefs.

In terms of changing or challenging teaching beliefs, it seems that professional development programs are facing an enormous task when it comes to how the program *could* serve to shape the curriculum. Some teachers want the mathematics curriculum handed to them, and these teachers seem to have a harder time adjusting what they've learned in Algebraic Thinking. They have grown comfortable with the given curriculum and perhaps somewhat complacent about teaching mathematics. There also appear to be teachers who are searching for anything they can find to keep their students busy (regardless of the fit between activities and the broader learning goals):

I have used it primarily to supplement what I'm already using (Investigations). At this grade level, you have to keep the children constantly busy, so I use whatever I can get. And the kids really like using manipulatives." (School 1)

And there are teachers who have changed the ways in which they think students ought to learn mathematics — and the ways in which they think they ought to approach the teaching of mathematics:

Algebraic Thinking is supposed to encourage higher order thinking. Rather than just paper and pencil, it is about trying to get kids to do it — they have to engage in writing about the procedures they use. We want kids to have different strategies to solve problems. They need to see that they may not always come up with the right answer, but they can see and explain the process. (School 2)

One teacher who was particularly enthusiastic about implementing Algebraic Thinking in the 5<sup>th</sup> grade classroom taught a lesson based on Algebraic Thinking during our observations. Prior to the lesson the teacher related what the lesson would be and why it was of importance. Although the lesson would be a review, the teacher worried that the mathematical concepts being taught (multiplicative properties of whole numbers) had not been learned, that it was crucial students learn this material in order to advance to more difficult concepts, and that prior presentations of the material may have been misdirected or may not have reached all of the students (e.g., the teacher was presenting the concepts in new ways). Although the lesson was highly structured, involved very little student-student interaction, and seemed to be centered on finding single solutions through guessing strategies, the students were enthusiastic participants in the lesson and their voices were clearly respected. Students were encouraged to take chances, to make educated guesses about solutions, and to help one another on a few occasions. Many of the students in the classroom were challenged by the problems presented and the teacher questioned students about how they arrived at their answers — what process they had undertaken to get to solutions. Although we cannot conclude that this teacher did not engage in similar practices prior to enrolling in the Summer Algebraic Thinking series, it was clear that the mode of thinking and pedagogy urged by Algebraic Thinking worked well with his own educational philosophy and appeared to work well with his students.

## ***Algebraic Thinking as a Supplement to Existing Curriculum***

In our conversations with teachers from all sessions, it was clear that they have used what they learned in Algebraic Thinking in different ways. Many participants stated that with the district adopting Investigations as its required mathematics curriculum, they have been able to use Algebraic Thinking as a supplement. Several participants provided their opinions on how they felt Algebraic Thinking compared to the required curriculum. Teachers provided a number of different opinions about this, ranging from seeing the two programs as almost indistinguishable to perceiving one as being formulaic and the other as being ambiguous to the point of lacking the structure necessary to implement it:

Both help kids think on a higher level but Investigations doesn't have enough ideas on its own — there is just not enough. Teachers have to go out and find other things — there are no workbooks/worksheets or consumables — things that teachers like to have. There is a place for a program like Algebraic Thinking. (1997–1998 participant)

Algebraic Thinking dealt more with computation — not just rote memory but visualizing numbers and amounts and manipulating numbers and amounts... Algebraic Thinking is more aimed at figuring out and as such it provides a broader variety of methods. On the other hand, Investigations stresses more “think math” — methods of problem solving, geometry and measurement. [Investigations] does not require teachers to use their imagination and essentially relegates them to the role of facilitators, which is just one way of teaching or one way of presenting it. (1997–1998 participant)

The programs are similar in that they are both student-centered [and both] allow students to set the pace for their own learning. (School 1)

## **CONCLUSIONS AND RECOMMENDATIONS**

Ultimately, algebraic thinking programs aim at changing thinking about mathematics and mathematics pedagogy. However, it would be impossible to understand the capacity of the two site schools without understanding the context in which these teachers reside.

### **Political Context: Testing and Assessment As An Obstacle**

Given the existing political climate surrounding assessment, standards, and accountability, it was not surprising that the group of teachers from the Summer 2000 session (as well as participants from prior years' sessions) were concerned with how Algebraic Thinking could support their students' performance on state tests that are tied to school accreditation. Moreover, given that poor districtwide test performance prompted an improvement plan which linked increased performance with increased testing of student progress (every 5 weeks), the stakes for standardized testing were elevated even higher for site schools. The 1997–1998 participants were able to provide some insight into how Algebraic Thinking supported their students' efforts on the state assessment (students of the Summer 2000 cohort have not taken the assessment):

Algebraic Thinking is new to the students but it helped with the [state test] — the students had some knowledge of what was going on [on the standardized test]. (1997–1998 participant)

I used Algebraic Thinking extensively at first when I was trying to come up with something different for the [state] test. (1997–1998 participant)

Reviewing notebooks of teachers who had participated in 1997–1998 series revealed that Algebraic Thinking participants were aware of the need to encourage explanations from their students. As one teacher poignantly stated in her notebook, “Our new state assessment focuses on the process and how students explain what they did to reach an answer. As a result there is a lot of work where the children must explain how they got an answer.” In this respect, Algebraic Thinking serves the state assessment well. However, there exists a danger that programs such as Algebraic Thinking will be primarily used toward the end of getting students to pass assessments rather than toward changing thinking about mathematics and building a solid foundation for future mathematics success.

### **Policy Context: Standardized Curriculum As An Obstacle**

Many participants from the 1997–1998 series had mixed reactions to Investigations. Some indicated that they felt the program was “being pushed down [their] throats”; others expressed concern that our evaluation efforts were part of data collection that would be used to again change the required mathematics curriculum (resulting in their changing existing practice again). At least one teacher expressed exasperation that teachers were not consulted about what the mathematics curriculum ought to look like: “the district [does] things without asking teachers.” Another teacher commented that she would be happy if the district would simply make a decision and stick to it: “[The] school district keeps changing from program to program. . . The board needs to be specific about what should be used.” (1997–1998 participant)

Almost all summer session participants stated that they planned on using Algebraic Thinking in their classrooms the following school year, though some were more enthusiastic than others about this possibility. Despite their initial commitment to teaching algebraically, from our observations and our second conversations, teachers seemed to be having a difficult time utilizing the Algebraic Thinking approach and could not articulate how it may have changed the way they think about mathematics. One of the obvious obstacles teachers face in implementing what they have learned in a program such as Algebraic Thinking is that they perceive their options as choices that must be made. For example, either Algebraic Thinking is integrated into the existing curriculum (as a supplement to Investigations) or entirely alter the existing curriculum (replace what is in use). Part of the reason for this either/or mentality is grounded in the focus on utility: “How can I use the materials in the classroom?” rather than, “how can I think about mathematics in a different way” (as more open-ended, constructive, etc.). This myopic focus seems to be a result of state and district policies surrounding mathematics curriculum and statewide testing. Despite the fact that participants may be inclined to use what they have learned in the Algebraic Thinking program, and remain impressed with the teaching style of the instructor or the capacity to change the ways



they think about mathematics, this is irrelevant once one realizes that teachers “are being told [through policy] how to teach.” (1997–1998 participant)

### **Algebraic Thinking, State Assessment, and Mandated Curricula**

Although there appear to be commonalities between Algebraic Thinking and Investigations, the reality for the teachers was that Algebraic Thinking could only be seen as a supplement to their existing (mandated) curriculum. All of the teachers we observed and interviewed suggested that if they were to use Algebraic Thinking, it would only be as a supplement to what they were already using. As such, it was apparent that participants considered their participation and the program as a whole in terms of its instrumental use, typically seeking out lessons and activities that might prove useful for preparing students to take the statewide assessment.

It is relatively clear that the amount of time that teachers devote to statewide tests (as a result of shifting toward increased accountability) results in relegating the importance of being better practitioners — being better at teaching, finding new ways of thinking about how we engage students and how to reach individual learners—to a secondary concern at best. And although some teachers did express concern about the amount of testing occurring in the schools, it was also evident that some teachers are unsure how they should react to the assessments and accountability culture of the poor urban district. For instance, a special education teacher stated that she was concerned that her students were being over-tested and that the standardized tests were inappropriate for children with special needs. Nevertheless, she recommended that programs such as Algebraic Thinking offer additional approaches or lessons that would help her children pass these assessments. Other participants expressed concern that the district “want[s] you to teach just what’s on that [state] test” and the policy decision at the district level to assess students every five weeks to determine their mastery of [state assessment] related instruction seems to give credence to this concern.

### **Additional Obstacles to Rethinking Mathematics and its Pedagogy**

Although several teachers were able to articulate the ways in which the Summer Algebraic Thinking series had an impact on their own thinking and practice, given the limited contact that participants had with the program, it is difficult to make even weak causal claims about this impact. This does not suggest, however, that program participants did not make changes to their pedagogy or alter the ways they think of mathematics. Rather, it suggests that the connections between participation in a brief professional development series and teaching remain tenuous. Indeed, we must ask how feasible it is to expect teachers to embrace a different way of thinking about their own practice — a different way of thinking or abstracting about mathematics — when the infrastructure at their schools does not support this. Unfortunately in some cases, the role of professional development programs is simply to fill out the required paperwork and log the required number of contact hours.

Considering professional development programs as instrumental at best (and as a necessary evil at worst) gives administrators the latitude they need to justify actions that undermine efforts (e.g., shortening actual time in professional development, disengaging, downplaying the importance of such activities). It seems obvious that organizations providing professional development are often fighting an uphill battle

against local administrators — the functionaries charged with enabling the programs within individual schools. In failing to see value in professional development programs or in dismissing these as something that has to be done simply because it is required, they (consciously or unconsciously) undermine the efforts of the sponsoring organizations and, perhaps most significantly, the efforts of those teachers in their school who are attempting to take something away from it.

A related notion is that teachers are not provided enough time to implement or engage in what they have learned. If we consider research into or reflection on one's own practice an important way to inform thinking about how to proceed, the lack of support within the schools for this activity seems problematic. Indeed, the assumptions that underlie evaluative activities such as this one are also problematic, since we assume that teaching practice ought to change as a consequence of participation and that students will benefit from these changes. Without allowing teachers time to reflect on what they need to do to impact the mathematics education of their students — by implicitly expecting that pedagogy and habits of mind will be altered in the course of a few short weeks — seems carelessly optimistic and ignores the exigencies of the particular situations (at the school level, at a broader policy level) in which participants are found.

Our follow-up telephone conversations with participants led us to the conclusion that, although teachers are provided time to participate in a number of professional development activities (ostensibly) to improve their practice, they are often expected to take their own time to change their curriculum and their teaching habits. With little guidance beyond the structures established at the policy level (e.g., school requirements to help meet building goals, district requirements to utilize a particular curriculum, state assessments and their implicit requirements), teachers are left with a sense of ambiguity around how they might use what they have learned. Teachers may be sent the message that engaging in reflection — how what was offered in a given professional development series fits into their existing curriculum or how it ought to change their pedagogy — ought to be done on their own time. And from our conversations with the participants, time is in short supply. Professional development activities that are expected to take root need to provide some support for teachers' efforts and time in adjusting curriculum, otherwise teachers end up embracing whatever methods and lessons seem easiest because they lack sufficient time and resources to do so.

### **Rethinking Professional Development**

These factors may indicate that educators committed to improving teaching need to re-think how professional development is conceived. Perhaps more emphasis should be placed on the sustainability of personal development among teachers and administrators, schoolwide development, and creating learning communities across school districts. It is unfortunate that for teachers and administrators, many reform efforts and their associated professional development activities begin to resemble each other — lacking local relevance and often marred by numerable failures when assessed in view of student performance. Therefore, professional development activities are often seen as “more of the same” and even things with the potential to facilitate change are overlooked. This was perhaps best captured by a teacher who had participated in Algebraic Thinking sessions prior to the Summer 2000 series. “In general, teachers see programs such as Investigations as ones that “peter out or fade away... Last year we

had another program on how to teach mathematics... Teachers went to workshops and then it was gone. Nothing lasts and the problem is that the district [does] things without asking teachers."

Perhaps what is necessary is a sustained commitment from districts and schools who are involved in professional development. This commitment should be evidenced efforts to ensure that schools and teachers participate as planned and that the choices of professional development opportunities offered are given serious consideration rather than being undertaken merely to meet district requirements for staff development.

It is unclear how many teachers are actually able to follow through with implementing a program such as this, given other exigencies within their state, district, and school. Moreover, sufficient contact time with the program seems crucial to increasing not only its capacity but also its efficacy. Explanations of what schools must commit to doing and how and why compliance with these commitments is paramount must precede all efforts.

In addition, it is quite possible that efforts to find common ground among required curricula, statewide tests, and professional development opportunities will be necessary first, if we expect a fit between our expectations of teachers and the professional development opportunities provided. Teachers need to understand why changing their practices will improve the chances that their children will have an opportunity to learn (in this case, higher, more abstract levels of mathematics). Otherwise, efforts at broad change are likely doomed from the start since they will tend to be subverted.

Changing the way teachers think about their teaching philosophy and pedagogical practice in an effort to better accommodate mandated mathematics curricula and state assessment methods is a daunting task. It will require considerable conversation about why changes are desirable and how the changes will influence the educational outcomes set for all students.



## APPENDIX A: PHASE I SURVEY

# Algebraic Thinking Questionnaire

## Part I. Demographic Information

(1) Gender (check one):

- ☐ Male  
☐ Female

(2) Ethnicity (check one):

- ☐ American Indian or Alaskan Native  
☐ African American  
☐ Asian or Pacific Islander  
☐ Hispanic  
☐ White (not of Hispanic origin)  
☐ Other

## Part II. Background Questions: Teaching and Teaching Mathematics

(1) How many years have you taught at either the elementary or secondary level prior to this school year? (check one)

- ☐ Fewer than three years  
☐ 3 - 5 years  
☐ 6 - 10 years  
☐ 11 - 15 years  
☐ 16 - 20 years  
☐ More than 20 years

(2) Are you currently a (check one):

- ☐ Self contained classroom teacher  
☐ Title I resource teacher  
☐ Math resource teacher  
☐ Other (please specify): \_\_\_\_\_

(3) What grade levels do you teach or serve as a resource to? \_\_\_\_\_

\_\_\_\_\_

(4) In what grade levels have you taught mathematics? \_\_\_\_\_

\_\_\_\_\_

(5) How many minutes do you spend teaching mathematics on a typical day? \_\_\_\_\_ (minutes)

(6) Approximately how many mathematics courses have you taken (including undergraduate and graduate courses)? \_\_\_\_\_

## Part III. Professional Development

(1) During the last two years, how much time have you spent on professional development in mathematics or the teaching of mathematics? (Include attendance at professional meetings and conferences, workshops, and courses. Do not include participation in Algebraic Thinking) (check one)

- ☐ None  
☐ Less than 6 hours  
☐ 6 - 15 hours  
☐ 16 - 35 hours  
☐ More than 35 hours

(2) Not including Algebraic Thinking, when was your most recent course or in-service education experience in mathematics or mathematics teaching? (check one)

- ☐ Within the last 3 months  
☐ 3 - 6 months ago  
☐ 7 - 12 months ago  
☐ 1 - 3 years ago  
☐ More than 3 years ago

#### Part IV. Pedagogy and Curriculum:

(1) Please answer the following questions with respect to teaching mathematics [circle the number that corresponds to your response]:

	Strongly Disagree	Somewhat Disagree	Neutral	Somewhat Agree	Strongly Agree
I am able to phrase questions to encourage more open-ended investigations	1	2	3	4	5
I use textbooks as a resource rather than as the primary instructional tool	1	2	3	4	5
I am able to manage a class of students who are using manipulatives	1	2	3	4	5
I use cooperative learning groups in mathematics instruction	1	2	3	4	5
I use a variety of alternative assessment strategies	1	2	3	4	5
My teaching of mathematics promotes investigation and growth of mathematical ideas in my students	1	2	3	4	5
I am able to make connections between what I teach and the previous and developing knowledge of my students	1	2	3	4	5
My mathematics lessons encourage mathematical reasoning	1	2	3	4	5
It is more important for students to engage in the process of conjecturing, inventing, and problem solving rather than focusing on finding right answers	1	2	3	4	5

(2) How comfortable do you feel teaching the following: [please circle the number that corresponds to your response]

	Not at all comfortable	Somewhat uncomfortable	Comfortable	Very Comfortable
Estimation	1	2	3	4
Number sense and numeration	1	2	3	4
Computation	1	2	3	4
Geometry	1	2	3	4
Measurement	1	2	3	4
Fractions and decimals	1	2	3	4
Patterns and relations	1	2	3	4
Number theory concepts	1	2	3	4

(3) About how often do students in your class do the following types of activities? (place a mark in the box that corresponds to your response for each item)

	Almost every day	Several times a week	About once a week	Less than once a week	Never
Do mathematics problems from textbooks					
Do mathematics problems on worksheets					
Work in small cooperative groups					
Work on mathematics projects					
Work together on classroom problems					
Play mathematics games					
Listen and take notes during presentation by teacher					
Make predictions and explore possible methods to solve a mathematical problem					
Learn about mathematics through real-life applications					
Write their reasoning about how to solve a problem					
Use calculators					
Use computers					
Use manipulative materials					

## Part V. Algebraic Thinking

(1) Please specify the Algebraic Thinking class in which you were enrolled and the number of sessions you attended:

\_\_\_\_ I was enrolled in the January 1997-February 1997 class and  
 \_\_\_\_ I attended 1-2 sessions  
 \_\_\_\_ I attended 3-4 sessions

\_\_\_\_ I was enrolled in the November 1997-May 1998 class and  
 \_\_\_\_ I attended 1-3 sessions  
 \_\_\_\_ I attended 4-5 sessions

\_\_\_\_ I was enrolled in the October 1998-March 1999 class and  
 \_\_\_\_ I attended 1-3 sessions  
 \_\_\_\_ I attended 4-6 sessions

(2) Did you take Algebraic Thinking for credit? \_\_\_\_ yes \_\_\_\_ no

(3) Did you receive credit? \_\_\_\_ yes \_\_\_\_ no

(4) Did you do a research project? \_\_\_\_ yes \_\_\_\_ no

(5) Did you keep a journal for the class? \_\_\_\_ yes \_\_\_\_ no

(6) Did Clare Heidema visit your classroom? \_\_\_\_ yes \_\_\_\_ no

(7) Why did you enroll in an Algebraic Thinking course? \_\_\_\_\_

\_\_\_\_\_  
 \_\_\_\_\_

(8) Were you able to utilize any of the information you learned in the course? (check one)  
\_\_\_\_\_ YES (please answer 8a & 8b of this section) \_\_\_\_\_ NO (please answer 8c of this section)

(8a) If yes, what information have you used? \_\_\_\_\_

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(8b) If yes, how have you used this information? \_\_\_\_\_

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(8c) If no, why were you unable to utilize the information? \_\_\_\_\_

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(9) Did your thinking about algebra/mathematics change as a result of the Algebraic Thinking course? (check one)

\_\_\_\_\_ YES (please answer 9a of this section) \_\_\_\_\_ NO (go to item 10)

(9a) If yes, in what ways has your thinking changed? \_\_\_\_\_

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(10) Did your teaching of mathematics change as a result of the Algebraic Thinking course?

\_\_\_\_\_ YES (please answer 10a of this section) \_\_\_\_\_ NO (go to item 11)

(10a) If yes, in what ways has your teaching changed? \_\_\_\_\_

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(11) [For the following items, please circle the number that corresponds to your answer]

	Strongly Disagree	Somewhat Disagree	Neutral	Somewhat Agree	Strongly Agree
I feel that my students benefited from my participation in Algebraic Thinking	1	2	3	4	5
Algebraic Thinking deepened my knowledge about ways to make mathematics more accessible to my students	1	2	3	4	5
The instructor for Algebraic Thinking was knowledgeable about the topic	1	2	3	4	5
The instructor for Algebraic Thinking was approachable	1	2	3	4	5
The instructor for Algebraic Thinking encouraged me to think about mathematics in different ways	1	2	3	4	5
I would encourage other teachers to attend courses—i.e., Algebraic Thinking courses	1	2	3	4	5
Algebraic Thinking is a class that would be beneficial for teachers with a range of math abilities	1	2	3	4	5

(12) Think of the best professional development course you have participated in and assign it a 5. Based on that, what rating would you assign the Algebraic Thinking course you took? (circle one):

1      2      3      4      5

**Thank you for completing the survey.**

To show our appreciation for your efforts, we will be sending you *Whelmers*, a McREL educational resource that you may share with your colleagues. The *Whelmers* CD-ROM, the first in McREL's Accessible Science Series, features 41 activities demonstrated by Steve Jacobs, science teacher and popular host of TV's "Jake's Attic." Each activity has been correlated by McREL with key sections of NRC's National Science Education Standards at the K-4, 5-8, and 9-12 grade levels.

## **APPENDIX B: GUIDING QUESTIONS—PHASE I (1997-1998 PARTICIPANTS)**

Background information on the teacher (grades taught, years in service)

In general, how would you characterize your teaching style? Is this consistent with how you teach mathematics/how would you characterize your approach to teaching mathematics?

What other kinds of professional development courses have you taken?

How was Algebraic Thinking comparable to other PD courses?

What kind of support do you have for taking PD courses — district, state requirements?

Were there other courses to choose from when you took Algebraic Thinking?

Were you able to utilize the information from the Algebraic Thinking course? What did you take away from the Algebraic Thinking program? How does this tie into your teaching of mathematics?

If McREL offers Algebraic Thinking again, what suggestions do you have for change?

## **APPENDIX C: PHASE II SURVEY (SUMMER 2000)**



Dear Algebraic Thinking Course Participant:

Please complete this survey and bring it with you to the first session of Algebraic Thinking. It will assist the instructor in better addressing your needs.

Thank you!

## Algebraic Thinking Questionnaire

- (1) How many years have you taught at either the elementary or secondary level prior to this school year? (check one)

☐ Fewer than three years  
☐ 3 - 5 years  
☐ 6 - 10 years

☐ 11 - 15 years  
☐ 16 - 20 years  
☐ More than 20 years

- (2) Are you currently a (check one):

☐ Self contained classroom teacher  
☐ Math resource teacher

☐ Title I resource teacher  
☐ Other (please specify): \_\_\_\_\_

- (3) What grade levels do you teach or serve as a resource to? \_\_\_\_\_

- (4) In what grade levels have you taught mathematics? \_\_\_\_\_

- (5) How many minutes do you spend teaching mathematics on a typical day? \_\_\_\_\_ (minutes)

- (6) Approximately how many mathematics courses have you taken (including undergraduate and graduate courses)? \_\_\_\_\_

- (7) During the last two years, how much time have you spent on professional development in mathematics or the teaching of mathematics? (Include attendance at professional meetings and conferences, workshops, and courses.) (check one)

☐ None  
☐ Less than 6 hours  
☐ 6 - 15 hours

☐ 16 - 35 hours  
☐ More than 35 hours

Were any of these hours about the Investigations curriculum? ☐ Yes ☐ No

If yes, how many? \_\_\_\_\_

- (8) Not including Algebraic Thinking, when was your most recent course or in-service education experience in mathematics or mathematics teaching? \_\_\_\_\_

- (9) Please answer the following questions with respect to teaching mathematics [circle the number that corresponds to your response]:

	Strongly Disagree	Somewhat Disagree	Neutral	Somewhat Agree	Strongly Agree
I am able to phrase questions to encourage more open-ended investigations	1	2	3	4	5
I use textbooks as a resource rather than as the primary instructional tool	1	2	3	4	5
I am able to manage a class of students who are using manipulatives	1	2	3	4	5
I use cooperative learning groups in mathematics instruction	1	2	3	4	5
I use a variety of alternative assessment strategies	1	2	3	4	5
My teaching of mathematics promotes investigation and growth of mathematical ideas in my students	1	2	3	4	5
I am able to make connections between what I teach and the previous and developing knowledge of my students	1	2	3	4	5
My mathematics lessons encourage mathematical reasoning	1	2	3	4	5
It is more important for students to engage in thinking about solving problems than to focus on finding right answers	1	2	3	4	5

(10) How comfortable do you feel teaching the following [please circle the number that corresponds to your response]:

	Not at all comfortable	Somewhat uncomfortable	Comfortable	Very Comfortable
Estimation	1	2	3	4
Number sense and numeration	1	2	3	4
Computation	1	2	3	4
Geometry	1	2	3	4
Measurement	1	2	3	4
Fractions and decimals	1	2	3	4
Patterns and relations	1	2	3	4
Number theory concepts	1	2	3	4

(11) About how often do students in your class do the following types of activities? (place a mark in the box that corresponds to your response for each item)

	Almost every day	Several times a week	About once a week	Less than once a week	Never
Do mathematics problems from textbooks					
Do mathematics problems on worksheets					
Work in small cooperative groups					
Work on mathematics projects					
Work together on classroom problems					
Play mathematics games					
Listen and take notes during presentation by teacher					
Make predictions and explore possible methods to solve a mathematical problem					
Learn about mathematics through real-life applications					
Write their reasoning about how to solve a problem					
Use calculators					
Use computers					
Use manipulative materials					

(12) Have you introduced any material into your mathematics lessons from Investigations? \_\_\_\_ Yes \_\_\_\_ No

If yes, please describe. \_\_\_\_\_

(13) Do you have or have you had the following concerns about implementing Investigations

I don't feel I am sufficiently familiar with Investigations. ; \_\_\_\_ Yes \_\_\_\_ No

I don't feel sufficiently prepared to teach Investigations. ; \_\_\_\_ Yes \_\_\_\_ No

I don't feel I have the materials I need to teach Investigations. ; \_\_\_\_ Yes \_\_\_\_ No

Do you have other concerns about implementing Investigations that the course could assist with?

\_\_\_\_ Yes \_\_\_\_ No If yes, please describe. \_\_\_\_\_

## APPENDIX D: ALGEBRAIC THINKING CONVERSATION (SUMMER 2000)

What grade level(s) will you be teaching next year?

What aspects of the Algebraic Thinking program did you like or were most appealing?

What didn't you like about the program or what aspects of the program (if any) would you change?

What do you feel like you learned from this program?

*For classroom teachers:* Do you think you will use what you have learned in Algebraic Thinking in your class next year?

*For others:* In what ways do you think your participation in Algebraic Thinking will change what you currently do?

*For classroom teachers only:* Would you be willing to participate in the follow-up study (which includes two additional conversations and a classroom visit)? McREL is able to offer a \$150.00 stipend for your participation. If willing to participate, we will send you a postcard toward the beginning of the school year to remind you that we will be contacting you about a classroom visit. We will then contact you to determine a time and day when you would be teaching mathematics and would be available for the observation. Then we'll send another reminder shortly before we come to visit that specifies the time and date of the visit and first follow-up conversation...

## APPENDIX E: FOLLOW-UP PHONE CONVERSATION GUIDING QUESTIONS (PHASE II)

ID:      *Grade level(s) taught:*      *Date of follow-up interview:*

[The purpose of this final interview is to better understand your perceptions of the teaching of mathematics and how (or whether) you think teaching mathematics could be served by programs such as Algebraic Thinking. I'm going to start the interview with some broad questions about the challenges of teaching mathematics and then move into more specific questions about Algebraic Thinking. While we are talking, I'd like to take some notes on the feedback you are providing, but as we have said, nothing you say to us will be associated with your name—in other words, the feedback you give us will be kept in the strictest of confidence. If that is okay with you, let's get started. The interview should take about 25-30 minutes.]

### General questions

- (1) Which of the following have been or are currently concerns for you related to teaching mathematics in your classroom?

	Was in past	Is now
A. Your own knowledge of mathematics?		
B. Your preparation to teach today's math?		
C. Your ability to answer student questions?		
D. Your confidence in teaching math?		
E. Other		

(1b) Have you or do you think it would be possible to overcome this/these challenges? If so, how? (is there support that could be provided that isn't...) For each above—what has worked for you? What would you like to do/have more of?

### Algebraic Thinking questions

- (2) How would you characterize the purpose of the Algebraic Thinking sessions?

(3) What mathematics package do you currently do use (and whether they are satisfied with it ← may be able to get this without directly questioning).

(4) Have you been able to utilize any of what you learned in the Algebraic Thinking Summer session?

**(4a) If yes**, what have they used? In what ways did they use it? (probe on whether used to **supplement** or **replace** existing materials) **If no**, what has kept you from using it?

(5) Would you say the Algebraic Thinking experience has changed your teaching? If so, how? And if so, has it had any impact on student learning? And if so, what? (how do you know?)

(6) Given your teaching experience since the beginning of this school year, would you say there is anything additional that Algebraic Thinking could provide to support you in the classroom? (try to get teachers to be as specific as possible) If they can't think of anything or state they are satisfied with what was covered, probe on specific aspects of the Algebraic Thinking program they would like to see more of...



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EFF-089 (3/2000)